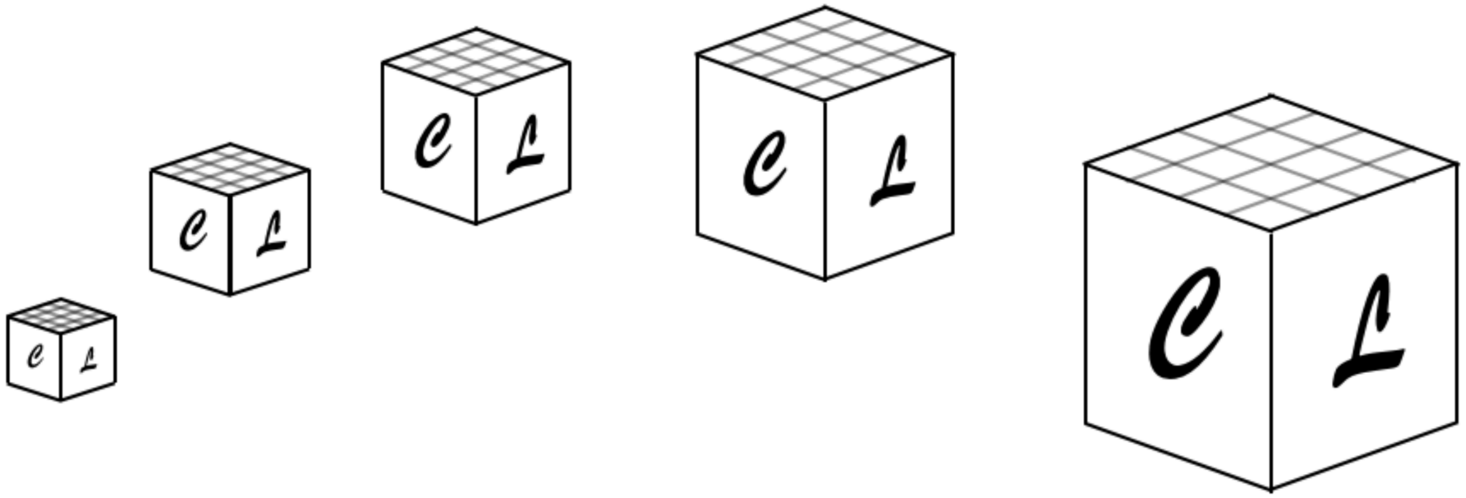
CosmoLattice

— School 2022 —



Simulating Non-minimally Coupled Scalars on the Lattice

Toby Opferkuch & Ben Stefanek

What are NMC Scalars good for?

$$\mathscr{L} \supset \xi R \phi^2 \propto \xi \boxed{H^2} \phi^2$$

Dominates only in the very early Universe

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Inflation & reheating:

Higgs inflation & reheating

Bezrukov & Shaposhnikov 0710.3755
Bezrukov et al 0812.3622
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$f(\phi)$ inflation & reheating

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De Felice & Tsujikawa 1002.4928

Geometric preheating

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Ricci reheating

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NMC reheating

Series of papers by DeCross et al
1510.08553, 1610.08868 & 1610.08916

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The Ricci Curvature Scalar

In post-inflationary FRW universe*

*For a single scalar dominating the energy density, recall also:

$$\rho \propto a^{-3(1+w)}$$

$$R = -3(1-3w)H^2$$

$$= \begin{cases} -12H^2\,, & \text{inflation } (w=-1) \\ -3H^2\,, & \text{matter domination } (w=0) \\ 0\,, & \text{radiation domination } (w=1/3) \\ 6H^2\,, & \text{kination domination } (w=1) \end{cases}$$

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Two extremely interesting cases:

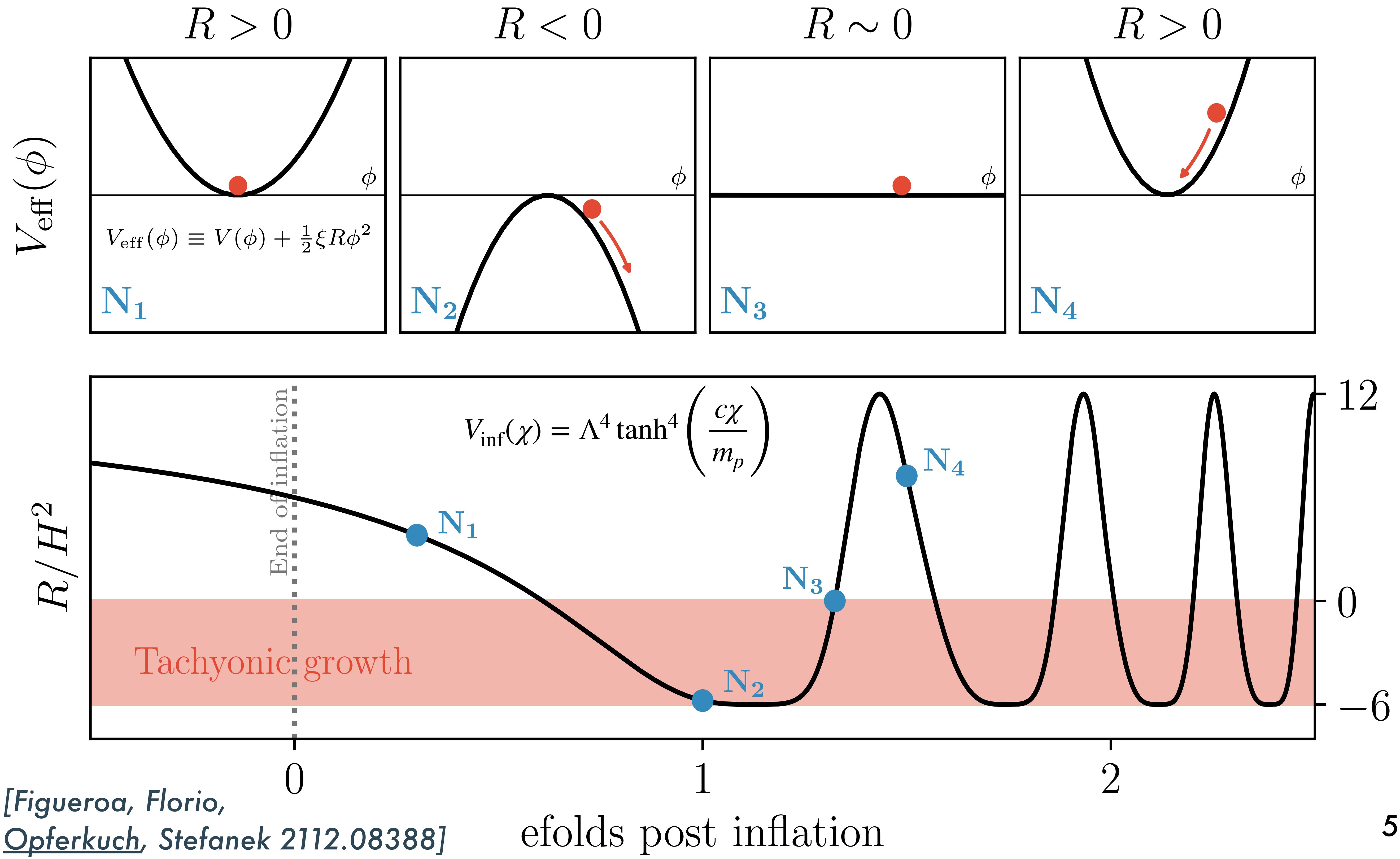
1. *R* changes sign if $w > 1/3$ after inflation $\Longrightarrow$ Ricci reheating

2. For an oscillatory post-inflationary inflaton potential $\Longrightarrow$ Geometric Preheating

$$V_{\text{inf}}(\chi) = \frac{1}{2}m^2\chi^2 \quad \Longrightarrow \quad R = -\frac{T}{m_p^2} \approx \frac{R_0}{4}\left(\frac{a_0}{a}\right)^3 [1 + 3\cos(2mt)]$$

Geometric Preheating

[Bassett & Liberati hep-ph/9709417]



[Figueroa, Florio, Opferkuch, Stefanek 2112.08388]

Initial Conditions

1. Solve inflationary EOS for background evolution:

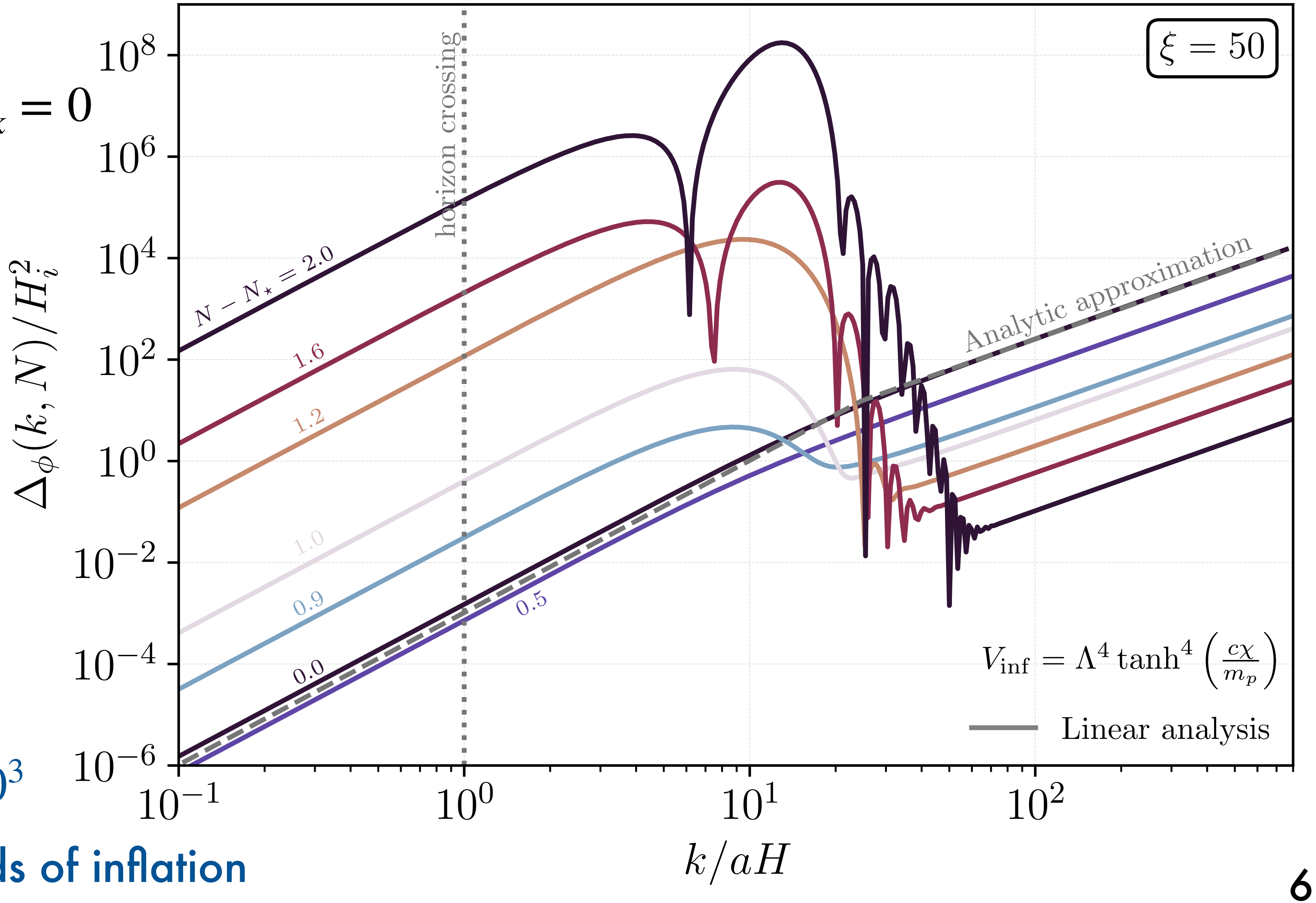
$$\ddot{\chi} + 3H\dot{\chi} + \frac{\partial V_{\rm inf}}{\partial \chi} = 0 \quad \text{where} \quad V_{\rm inf}(\chi) = \Lambda^4 \tanh^p\left(\frac{c\chi}{m_p}\right) \quad \text{and } p = 4{,}6$$

2. Solve mode equations initialising each mode in Bunch-Davies:

$$\ddot{\varphi}_k + H\dot{\varphi}_k + \left[\frac{k^2}{a^2} + \left(\xi - \frac{1}{6}\right)R\right]\varphi_k = 0$$

with initial conditions

$$\varphi_k(k/(aH) \gg 1) \approx \frac{1}{\sqrt{2k}} e^{\frac{ik}{aH}}$$



Numerical implementation:

Solved in cosmic time $(\alpha = 0)$

$k = 512$ log-spaced modes

Penetration factor $k/(aH) = \beta \sim 10^3$

Implies $\Delta N \sim \log(10^3\beta) \sim 14$ e-folds of inflation

Cosmology on the Lattice

System of equations:

$$\phi'' + (3-\alpha)\frac{a'}{a}\phi' - \frac{\nabla^2\phi}{a^{2(1-\alpha)}} + a^{2\alpha}\xi R\phi = -a^{2\alpha}\frac{\partial V}{\partial\phi}$$

NMC spectator field

$$\chi'' + (3-\alpha)\frac{a'}{a}\chi' - \frac{\nabla^2\chi}{a^{2(1-\alpha)}} = -a^{2\alpha}\frac{\partial V_{\rm inf}}{\partial\chi}$$

Inflaton

$$\frac{a''}{a} + (1-\alpha)\left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{6}R$$

FRW Background evolution

with the Ricci scalar:

$$F(\phi) \equiv \frac{1}{1+\left(6\xi-1\right)\xi\langle\phi^2\rangle/m_p^2}$$

$$R = \frac{F(\phi)}{m_p^2}\left[\left(6\xi-1\right)\left(\frac{1}{a^{2\alpha}}\langle\phi'^2\rangle - \frac{1}{a^2}\langle(\nabla\phi)^2\rangle\right) - 6\xi\langle\phi V_{,\phi}\rangle + 4\langle V + V_{\rm inf}\rangle - \frac{1}{a^{2\alpha}}\langle\chi'^2\rangle + \frac{1}{a^2}\langle(\nabla\chi)^2\rangle\right]$$

BUT: *R* depends on all fields and their conjugate momenta

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BUT: *R* depends on all fields and their conjugate momenta

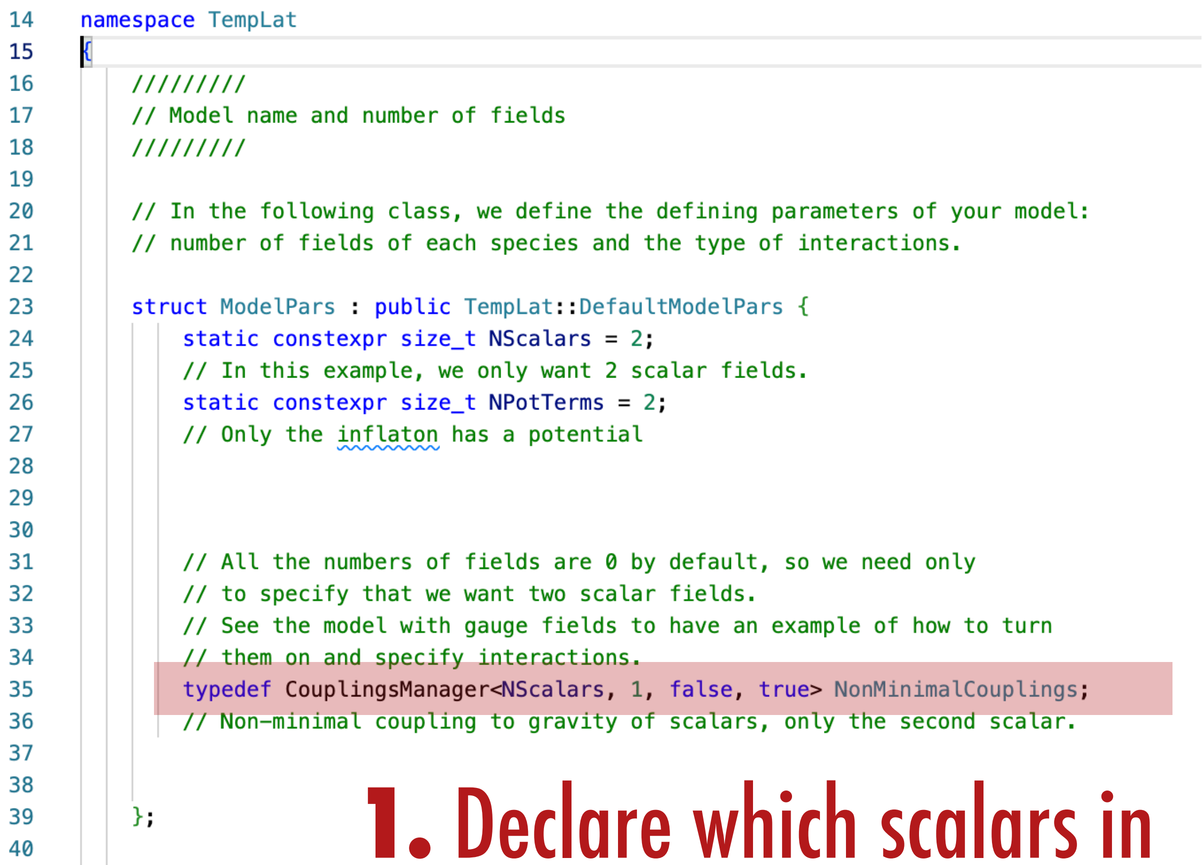
Cannot use symplectic integrators $\Longrightarrow$ Implicit Runge-Kutta

Public Implementation
in $\mathscr{C}$osmo$\mathscr{L}$attice

[Figueroa, Florio, Torrenti, and Valkenburg: 2006.15122]
[Figueroa, Florio, Opferkuch, Stefanek: 2112.08388]
[Figueroa, Florio, Opferkuch, Stefanek: To appear]

CosmoLattice NMC Inputs

Model file with NMC scalar (NMC_tan4.h):



```
namespace TempLat
{
    /////////
    // Model name and number of fields
    /////////

    // In the following class, we define the defining parameters of your model:
    // number of fields of each species and the type of interactions.

    struct ModelPars : public TempLat::DefaultModelPars {
        static constexpr size_t NScalars = 2;
        // In this example, we only want 2 scalar fields.
        static constexpr size_t NPotTerms = 2;
        // Only the inflaton has a potential



        // All the numbers of fields are 0 by default, so we need only
        // to specify that we want two scalar fields.
        // See the model with gauge fields to have an example of how to turn
        // them on and specify interactions.
        typedef CouplingsManager<NScalars, 1, false, true> NonMinimalCouplings;
        // Non-minimal coupling to gravity of scalars, only the second scalar.


    };

```

1. Declare which scalars in the theory are minimally versus non-minimally coupled

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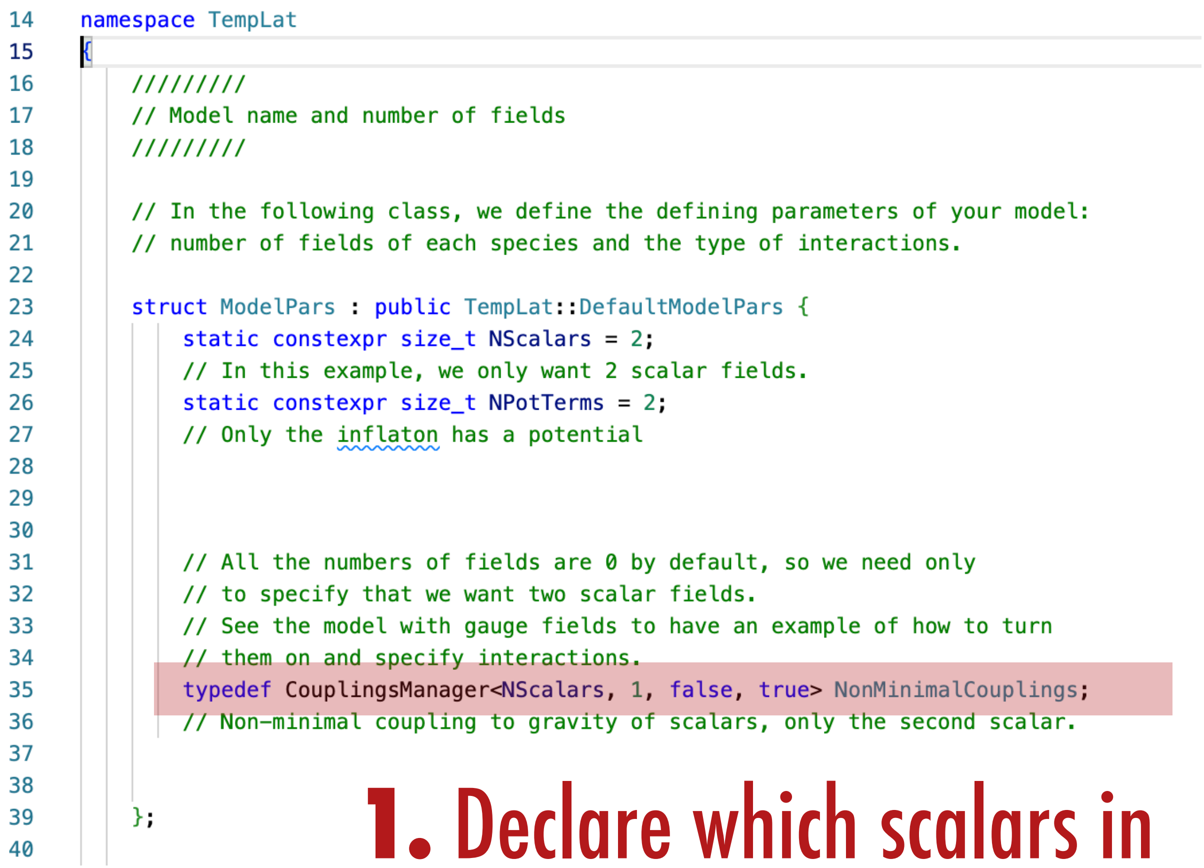

```cpp
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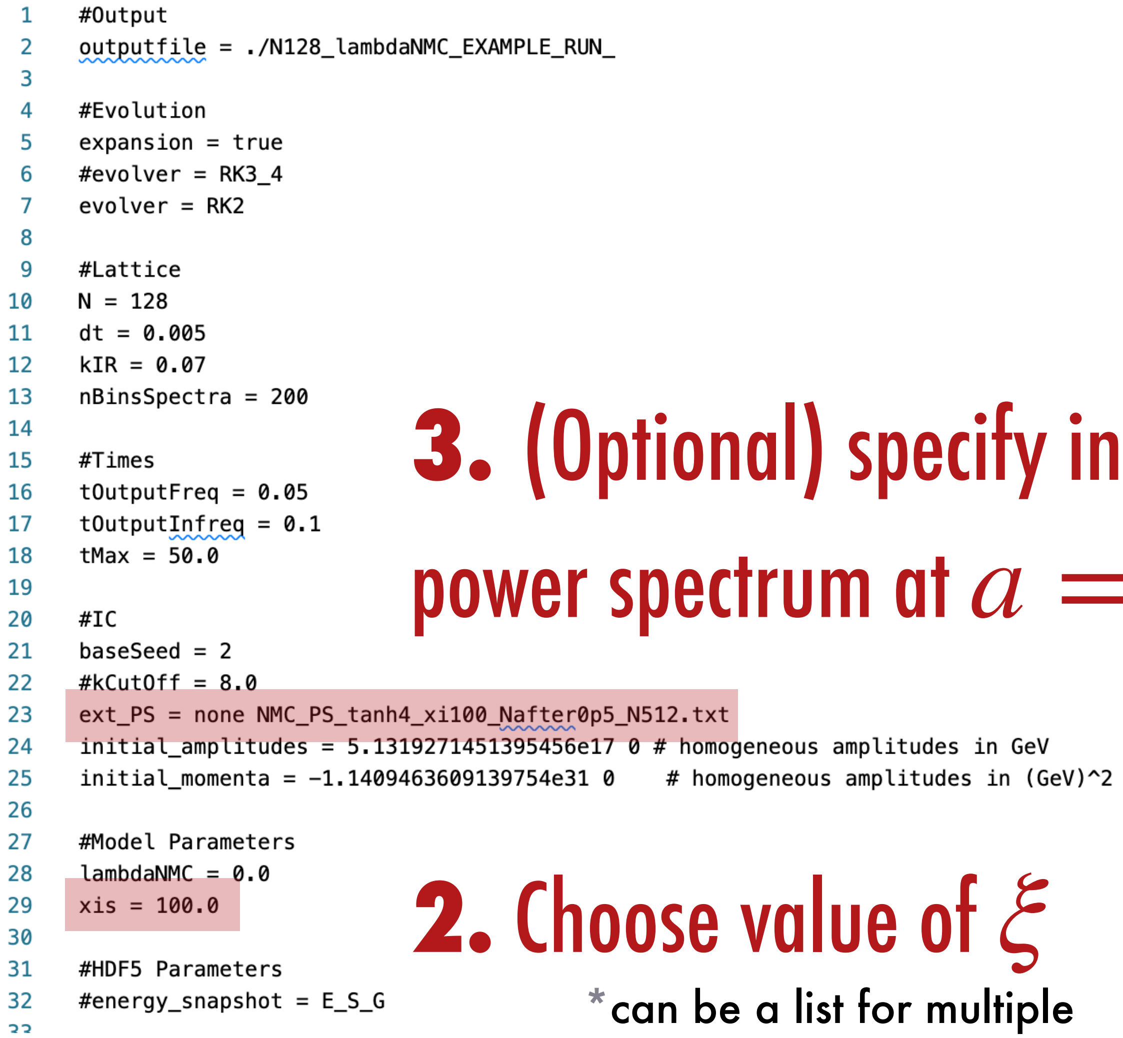

```
#Output
outputfile = ./N128_lambdaNMC_EXAMPLE_RUN_

#Evolution
expansion = true
#evolver = RK3_4
evolver = RK2

#Lattice
N = 128
dt = 0.005
kIR = 0.07
nBinsSpectra = 200

#Times
tOutputFreq = 0.05
tOutputInfreq = 0.1
tMax = 50.0

#IC
baseSeed = 2
#kCutOff = 8.0
ext_PS = none NMC_PS_tanh4_xi100_Nafter0p5_N512.txt
initial_amplitudes = 5.1319271451395456e17 0 # homogeneous amplitudes in GeV
initial_momenta = -1.1409463609139754e31 0    # homogeneous amplitudes in (GeV)^2

#Model Parameters
lambdaNMC = 0.0
xis = 100.0

#HDF5 Parameters
#energy_snapshot = E_S_G
```

3. (Optional) specify input power spectrum at $a = 1$

2. Choose value of ξ

*can be a list for multiple NMC scalar fields

CosmoLattice NMC Outputs

1. Output file for scale factor & R (*_average_scale_factor.txt):

Prints columns:

$$\tilde{\eta}, a, a', \frac{a'}{a}, R$$

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$$\tilde{\eta}, \langle\tilde{\phi}_n\rangle, \langle\tilde{\phi}'_n\rangle, \langle\tilde{\phi}_n^2\rangle, \langle\tilde{\phi}_n'^2\rangle, \mathrm{rms}(\tilde{\phi}_n), \mathrm{rms}(\tilde{\phi}'_n)$$

**Same as public version of CosmoLattice*

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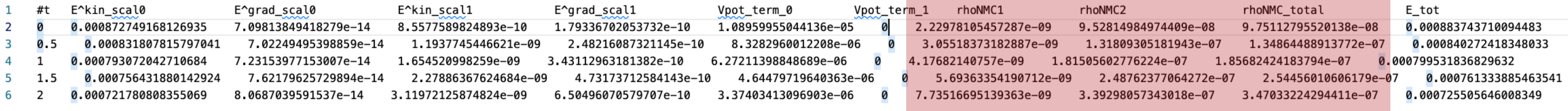
3. Output file for NMC scalar energy densities (*_average_energies.txt):



```
#t   E^kin_scal0            E^grad_scal0          E^kin_scal1           E^grad_scal1          Vpot_term_0           Vpot_term_1   rhoNMC1               rhoNMC2               rhoNMC_total          E_tot
0    0.000872749168126935   7.09813849418279e-14  8.5577589824893e-10   1.79336702053732e-10  1.08959955044136e-05  0   2.22978105457287e-09  9.52814984974409e-08  9.75112795520138e-08  0.000883743710094483
0.5    0.000831807815797041   7.02249495398859e-14  1.1937745446621e-09   2.48216087321145e-10  8.3282960012208e-06  0   3.05518373182887e-09  1.31809305181943e-07  1.34864488913772e-07  0.000840272418348033
1    0.000793072042710684   7.23153977153007e-14  1.654520998259e-09   3.43112963181382e-10  6.27211398848689e-06  0   4.17682140757e-09  1.81505602776224e-07  1.85682424183794e-07  0.000799531836829632
1.5    0.000756431880142924   7.62179625729894e-14  2.27886367624684e-09   4.73173712584143e-10  4.64479719640363e-06  0   5.69363354190712e-09  2.48762377064272e-07  2.54456010606179e-07  0.000761333885463541
2    0.000721780808355069   8.0687039591537e-14  3.11972125874824e-09   6.50496070579707e-10  3.37403413096903e-06  0   7.73516695139363e-09  3.39298057343018e-07  3.47033224294411e-07  0.000725505646008349
```

$$\rho_\chi(\eta) = \frac{1}{2a^{2\alpha}}\langle\chi'^2\rangle + \frac{1}{2a^2}\langle(\nabla\chi)^2\rangle + \langle V(\chi)\rangle$$

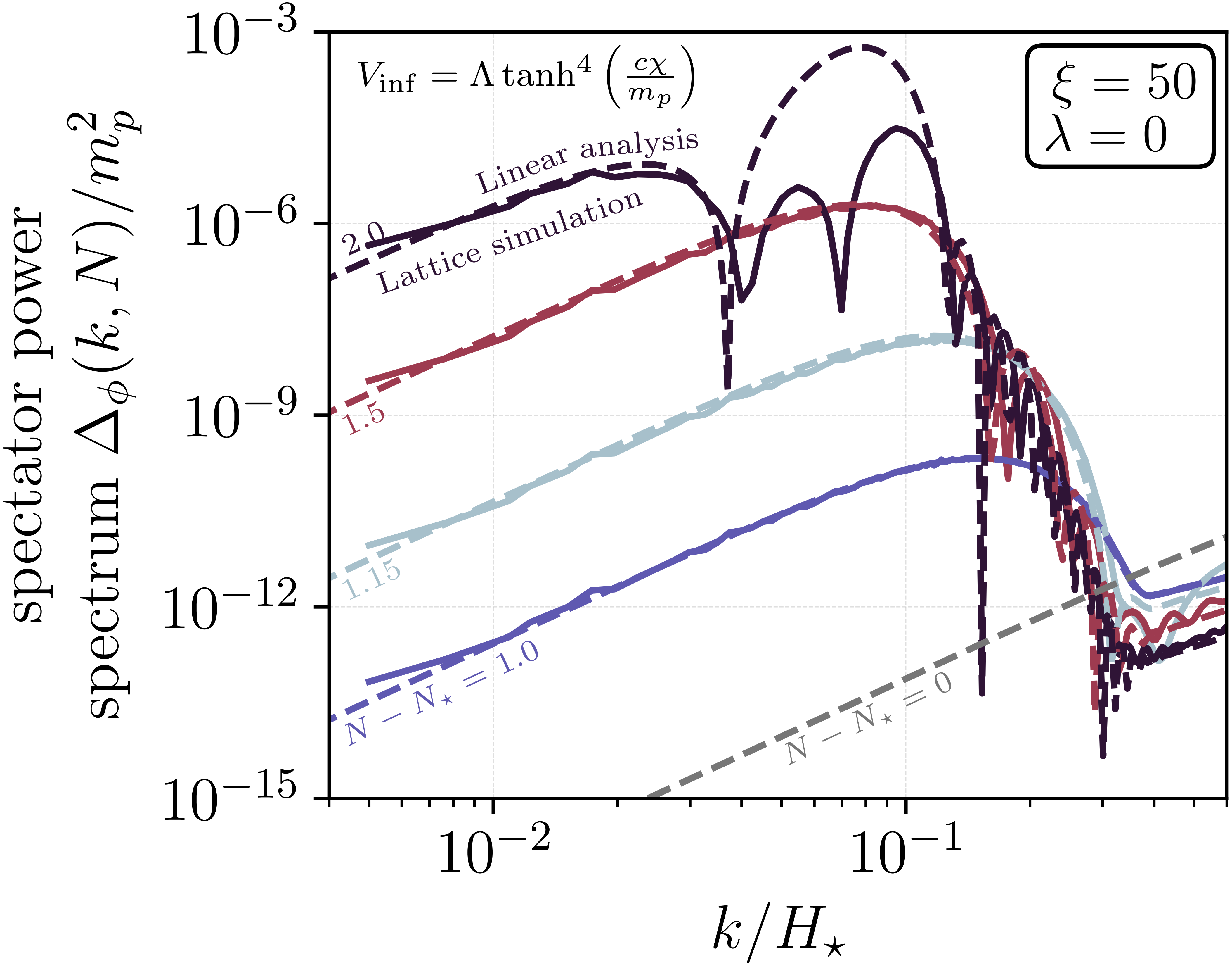
Inflaton field (scal0)

$$\rho_\phi(\eta) = \frac{1}{2a^{2\alpha}}\langle\phi'^2\rangle + \frac{1}{2a^2}\langle(\nabla\phi)^2\rangle + \langle V(\phi)\rangle$$

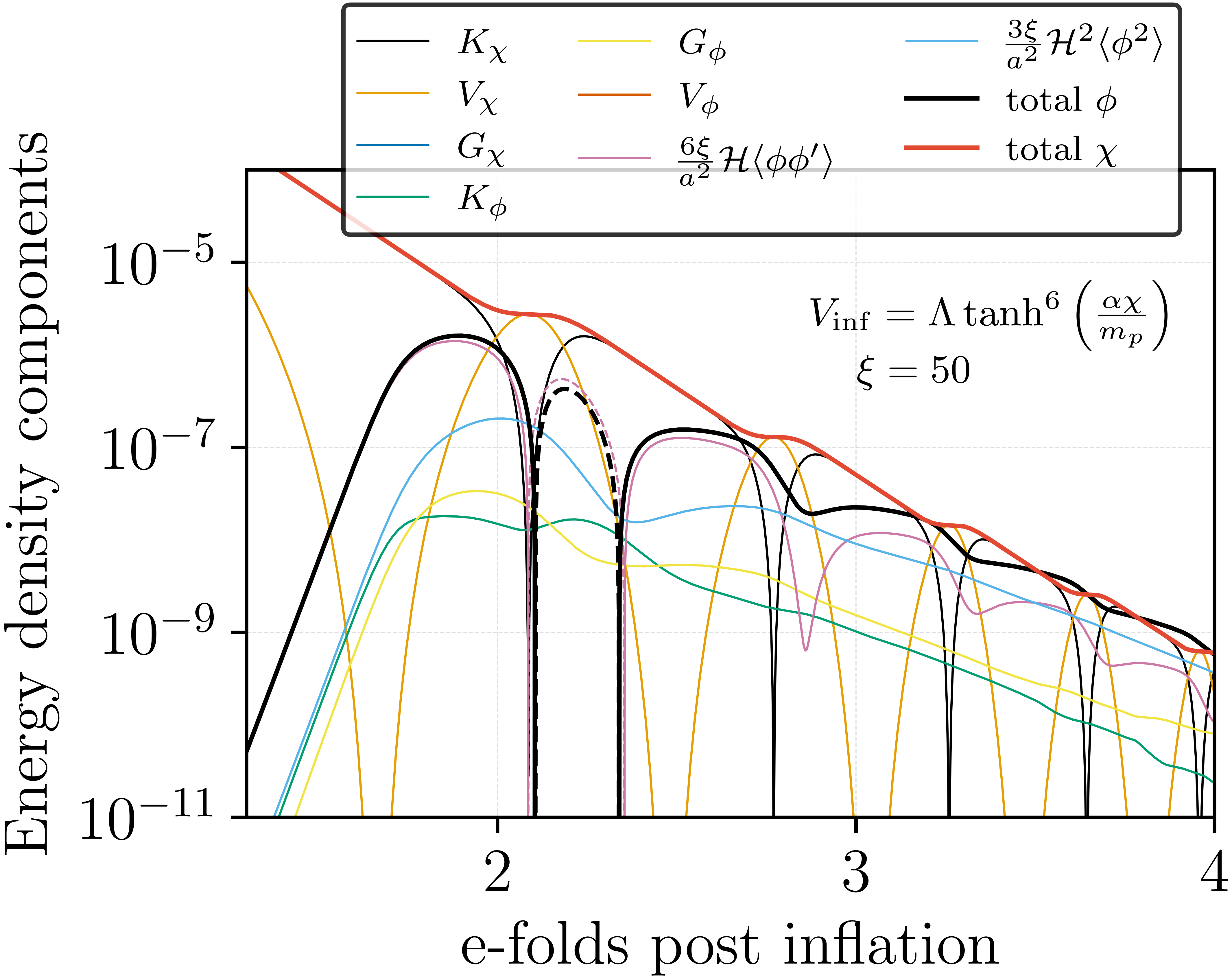
$$+ \overset{\text{rhoNMC1}}{\frac{3\xi}{a^{2\alpha}}\mathscr{H}^2\langle\phi^2\rangle} + \overset{\text{rhoNMC2}}{\frac{6\xi}{a^{2\alpha}}\mathscr{H}\langle\phi\phi'\rangle} - \overset{\langle\dots\rangle = 0}{\cancel{\frac{\xi}{a^2}\langle\nabla^2\phi^2\rangle}}$$

NMC spectator field (scal1)

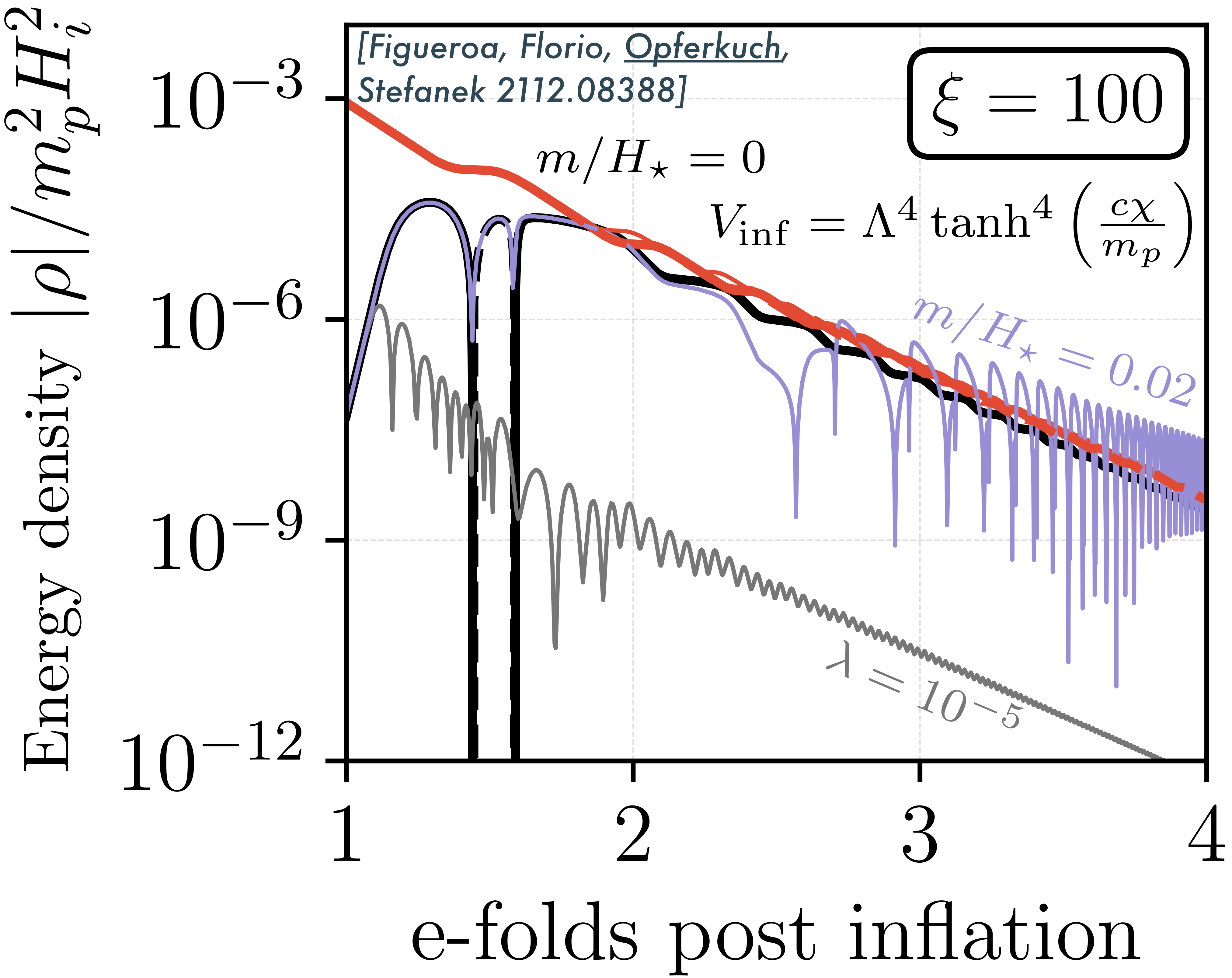
Power Spectrum



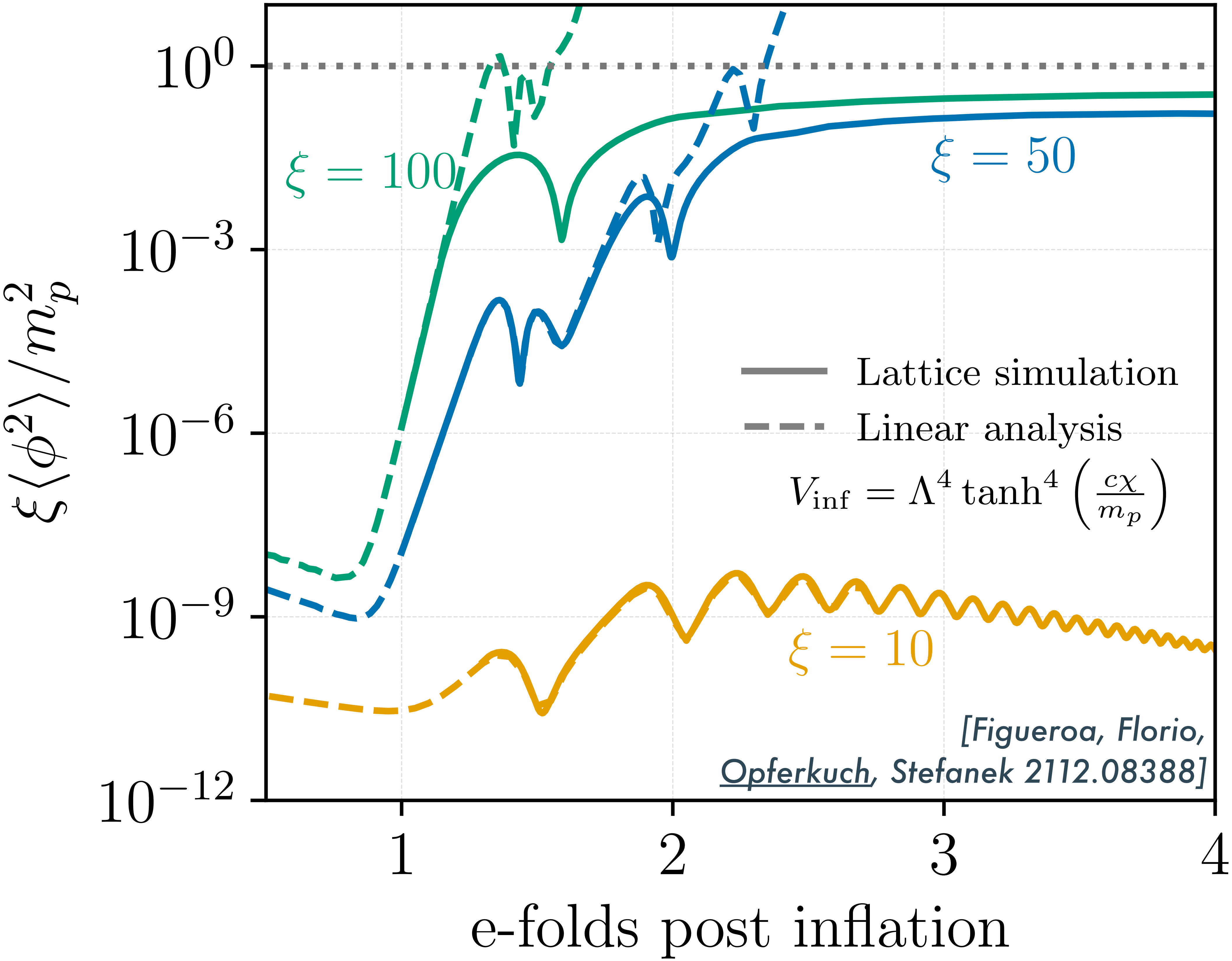
Energy density



Reheating Viability



Back-reaction Results



[Figueroa, Florio, Opferkuch, Stefanek 2112.08388]

Current/Future Directions

- Extend $\mathscr{C}\mathrm{osmo}\mathscr{L}\mathrm{attice}$ to include GWs in the presence of a dominant NMC background
- Simulate the full NMC Higgs scenario (including all relevant SM fields)
- Study constraints on NMC Higgs in the presence of an oscillating inflaton

Bounded range of allowed ξ for the operator $\xi R|H|^2$